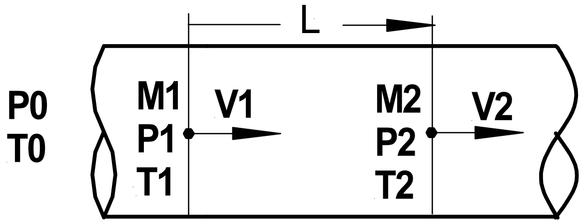
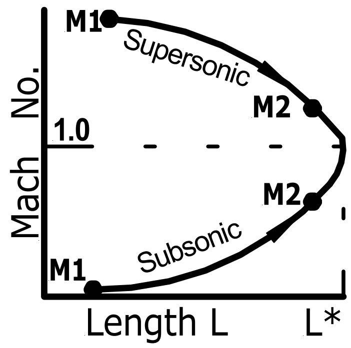
12 Fanno Flow (Adiabatic Flow with Friction)

This program calculates the exit properties of a compressible gas flowing adiabatically (no heat transfer) in a constant diameter, round duct with friction.





By trial and error the following Fanno equation is solved:

$$f\frac{L}{D} = \frac{1}{k}\left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right) + \frac{k+1}{2k}\ln\left[\left(\frac{M_1}{M_2}\right)^2 \frac{(k-1)M_2^2 + 2}{(k-1)M_1^2 + 2}\right]$$

Caution: as indicated in the above graph, the Fanno equation can have two solutions: a subsonic solution and a supersonic solution.
Although both solutions are valid, for the purposes of this program, we are concerned only with the subsonic region.

- **Therefore, to avoid finding the incorrect solution to the Fanno equation, the initial Mach No. (M1) must always be < 1.0, and the exit Mach No. (M2) must always be <= 1.0. The program warns when either of the above criteria is violated.**
- **To find the properties at the limiting sonic velocity condition, the program sets M2=1.0 and calculates the maximum flow, and the maximum length.**

For the initial position (1) and/or final position (2) the following values are calculated:

$$\sigma = \frac{P144(MW/1544)}{ZT} \qquad V = \frac{0.0509W}{\sigma d^2} \qquad V_S = \sqrt{kgT(1544/MW)}$$

$$M = V / V_S \qquad f = (0.25) / \left[\log_{10}\left(\frac{\varepsilon}{3.7D} \right) \right]^2$$

Notes:

- **The friction factor, f , is calculated in the fully turbulent region (i.e. independent of Reynolds no.). To change f , change the pipe roughnes, ε .**
- **To change the calculated <u>density</u>, change the compressibility factor, Z.**

The exit pressure and temperature are found from the following relationships:

$$\frac{T_1}{T_2} = \frac{1 + \frac{(k-1)M_2^2}{2}}{1 + \frac{(k-1)M_1^2}{2}} \qquad \frac{P_1}{P_2} = \frac{M_2}{M_1}\sqrt{\frac{T_1}{T_2}}$$

The stagnation values can also be found:

$$\frac{T_0}{T_1} = 1 + \frac{M_1^2(k-1)}{2} \qquad \frac{P_0}{P_1} = \left(\frac{T_0}{T_1} \right)^{k/(k-1)}$$

The compressible and incompressible pressure drops can be compared:

$$\Delta P_{comp} = P_1 - P_2 \qquad \Delta P_{inc} = 3.36x10^{-6}\left(\frac{fLW^2}{\sigma_1 d^5} \right)$$

Symbols

Symbol	Variable	Units
0	Stagnation Position	
1	Initial Position	
2	Exit Position	
C	constant	dimensionless
d	pipe diameter	in
D	pipe diameter	ft
f	friction factor	dimensionless
g	gravitational constant	ft/sec/sec
k	ratio of specific heats Cp/Cv	dimensionless
L	pipe length	feet
M	Mach No	dimensionless
MW	molecular weight	lb/lb mole
P	pressure	psia
t	temperature	F
T	temperature	R
V	velocity	fps
Vs	sonic velocity	fps
W	mass flow	lb/hr
Z	compressibility factor	dimensionless
ΔP	pressure drop	psi
ε	pipe roughness	feet
λ	specific Volume	cf/lb
σ	density	lb/cf

Reference:
McGraw Hill, Inc: FLUID MECHANICS, 8th ed, Victor L. Streeter &. Benjamin Wylie, section 7.6
CRANE: FLOW OF FLUIDS, Technical Paper No. 410