PIECEWISE PARABOLIC DATA INTERPOLATION

PPDI[tm] is a program that interpolates between user-specified data points. It constructs a parabola between each pair of points such that BOTH the values and the first derivative are continuous. By contrast, a least squares curve fit does not necessarily pass through any of the data points and piecewise linear interpolation does not offer continuous derivatives. This continuous derivative feature is particularly important in situations where sharp changes of direction are not appropriate, as in, for example, temperature vs. time applications. The outputs of the program are a set of coefficients for each parabola and, optionally, a set of interpolated values. If "n" data points are input, there will be "n-1" sets of coefficients. The coefficients or the interpolated values can then be input to another program as needed.

Installation

PPDI runs on any computer using a version of Windows[tm] 98, ME, XP, NT or 2000 capable of running 32-bit applications. You have received two files, the one you are reading as well as "PPDI Install.exe". Running 'PPDI Install' installs PPDI itself as well as a sample data file named "PPDI Sample.txt". The disk space required can vary greatly between 100KB and 4MB. This wide range is because PPDI was developed using Microsoft® software and the number of support files installed depends upon whether or not you have installed similar software in the past or the support files are already a part of your Windows installation.

By default, the program and the data file will be installed in the folder

C:\Program Files\PPDI

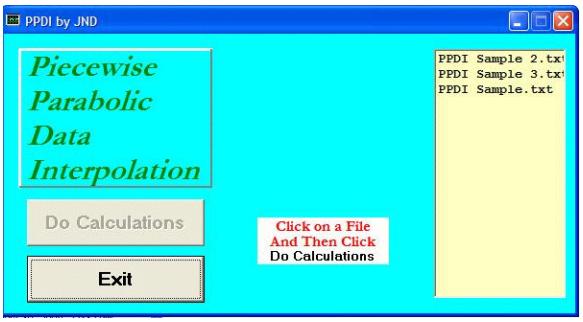
although you may change its location. Regardless, the program will then appear on the Start Menu, but no desktop icon will be installed.

INSTALLATION NOTE

> On some systems with large disk drives, the installation program may report that there is actually a negative amount of disk space available. It is safe to simply ignore this message and click on the "Install Now" button.

Input

PPDI's opening screen is shown below. The program reads user-prepared text files for its



data and a program such as Windows Notepad can be used to create them. A minimum data file consists of six lines total: four data lines, an error band line and an interpolation line. These last two types of input lines will be explained below.

Each file must have a ".txt" suffix and must be located in the same folder as the program. Each data line of the file should contain a pair of numbers separated by a comma in the form "X, Y". The constraints on the values are as follows:

- All values may be integer or floating point, though there should be no embedded commas.

- Values may be positive or negative as required, although the X-values must appear in increasing order.

- The Y-values need not monotonically increase, although they are shown this way in the sample file for clarity purposes.

- The X-values do not need to be equally spaced, though, once again, they are shown this way in the sample file.

Therefore, the first few lines of such a file might look like this:

```
3.5,17.25
9,45.125
12,114
17.5,202
   .
   .
   .
```

If it helps in readability when preparing the file, blank spaces may appear immediately before or after the comma. The maximum number of data points is limited only by the amount of computer memory available.

After the data lines, the next line of the file specifies whether an error band is to be used and of what type. This feature has been incorporated because, if the data is based upon instrument readings, there is often an associated error with each measurement. Usually this error is stated as a percentage of the reading or as a function of the instrument's full scale value. The benefit to the user of incorporating an error band is that smoother overall curves can be produced. The use of an error band relaxes the constraint that the resulting curves pass through each data point and replaces it with one that only specifies that the curves pass through each error band.

To use an error band for each point as a percentage of the Y-value, the error band line of the file should look as follows:

```
P=i
```

where "P" specifies a percentage error band and "i" is an integer percentage. For example, to specify an error band of plus-or-minus five percent, "i" should be "5". If no error band is desired, input a value of zero.

Alternatively, for an error band of a fixed amount at each point, make the line read:

```
A=x
```

where "A" specifies an absolute error band and "x" is a fixed amount to be applied to each Y-value in a plus-or-minus fashion. Again, a zero specifies no error band.

The last line of the file specifies how many interpolated values should be output. The interpolated points will have equally spaced X-values between and including the minimum X and the maximum X input by the user. The line should read as follows:

```
I=n
```

where "I" specifies interpolation and "n" is the number of points. Once again, the maximum number of points is limited only by computer memory. If no interpolated values are desired, input a value of zero.

The data files should be placed in the same folder as PPDI as stated previously. Upon running the program, you will be shown a list of ".txt" files from which to choose. There are no screen inputs other than clicking on a file name and clicking on two buttons, "Do Calculations" and "Exit", as appropriate.

Output

At each interval between pairs of data points, PPDI solves three equations in three unknowns to calculate the parameters "A", "B" and "C" for a parabola of the form "A*X^2 + B*X + C" using the coordinates of the two points and

the known derivative at one of the points. However, to get started with the first pair, a derivative is not known. Therefore, the program constructs a parabola through the first three points and then proceeds from there. This biases the results because the first two intervals are treated differently from the rest. To minimize this bias, PPDI makes the calculations in both "forward" and "reverse" directions, that is, from lowest X-value to highest and then highest to lowest. The final output is the average of the two solutions.

If error bands are specified, PPDI has multiple choices as to the parabola to use. It chooses the one with the lowest inflection (absolute value of "A") as this tends to yield the smoothest resulting series of curves.

There are no screen outputs from the program; all outputs are sent to the default printer, so it is best to choose the printer you wish to use before running the program.

If a given curve segment degenerates from a parabola to a straight line, the "A" value is shown as a blank instead of "0" for readability purposes. A sample output is shown at the end of this document, along with a graph showing the input data as well as the interpolated values.

See the box on the next page for a discussion of scaling input data when appropriate, as well as output significant digits and decimal places and making best use of them.

Error Messages

There is a series of error messages that can appear on the screen. A message can appear for any of the following reasons:

- The minimum number of data points has not been input.
- One of the last two lines has not been input in the proper format or is missing.
- The X-values are not in increasing order.

If the specific problem cannot be identified, the general purpose message ***Error Reading File***! will appear. In each case, you may choose another file by clicking on it or clicking on the "Exit" button.

Assistance

Should you encounter any problems or wish to suggest some additional program feature, please send an email to:

NickDavis@EarthLink.net

I'll try to respond as quickly as possible. I would also like to hear about interesting applications where you found PPDI to be useful.

A Note on Scaling, Significant Digits & Decimal Places

Double precision arithmetic was used throughout in the development of PPDI. Nevertheless, it is recommended that if your data values are large you scale them down before running the program. Likewise, if they have absolute values of less than one, scale them up as necessary. This will result in the greatest precision and best use of the available significant digits.

The data values will appear on the output showing the number of decimal places required to show the largest number of decimal places input. For example, if two Y-values of 1.25 and 2.375 are input, all data Y-values will be printed with three decimal places. Calculated Y-values, as with the interpolated values, will be shown with one more decimal place than was present in the input data, subject to the universal constraint of a maximum of four decimal places. If you wish to force the maximum number of decimal places, input at least one Y-value with four decimal places, even if the final few digits are zero. However, it is recommended that you try the automatic mode first to ensure that doing so will not result in an overflow of the nine space output field – seven significant digits plus the sign and decimal point. The "A", "B" and "C" coefficients are always output with two decimal places.

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D A T A

Point	X	Y-Lo	Y	Y-Hi	Point	X	Y-Lo	Y	Y-Hi
1	1	4.075	4.29	4.504	6	6	86.906	91.48	96.054
2	2	8.835	9.30	9.765	7	7	107.758	113.43	119.101
3	3	22.011	23.17	24.328	8	8	141.835	149.30	156.765
4	4	40.185	42.30	44.415	9	9	178.533	187.93	197.326
5	5	48.554	51.11	53.665	10	10	210.235	221.30	232.365

C O E F F I C I E N T S

	Forward			Reverse		
Points	A	B	C	A	B	C
1 to 2	3.28	-4.15	4.94	-26.04	82.44	-51.90
2 to 3	3.28	-4.15	4.94	37.20	-170.50	201.04
3 to 4	2.65	-0.37	-0.73	-36.84	273.70	-465.26
4 to 5	-7.34	79.55	-160.57	34.46	-296.65	675.45
5 to 6	27.10	-264.88	700.51	-14.70	194.93	-553.51
6 to 7	-28.15	398.10	-1288.42	8.73	-86.21	289.91
7 to 8	18.68	-257.53	1006.30		36.00	-137.82
8 to 9		41.42	-189.51		36.00	-137.82
9 to 10		41.42	-189.51		36.00	-137.82

	Average					
Points	A	B	C	Points	A	B	C
1 to 2	-11.38	39.15	-23.48	6 to 7	-9.71	155.94	-499.25
2 to 3	20.24	-87.32	102.99	7 to 8	9.34	-110.77	434.24
3 to 4	-17.09	136.67	-233.00	8 to 9		38.71	-163.67
4 to 5	13.56	-108.55	257.44	9 to 10		38.71	-163.67
5 to 6	6.20	-34.97	73.50				

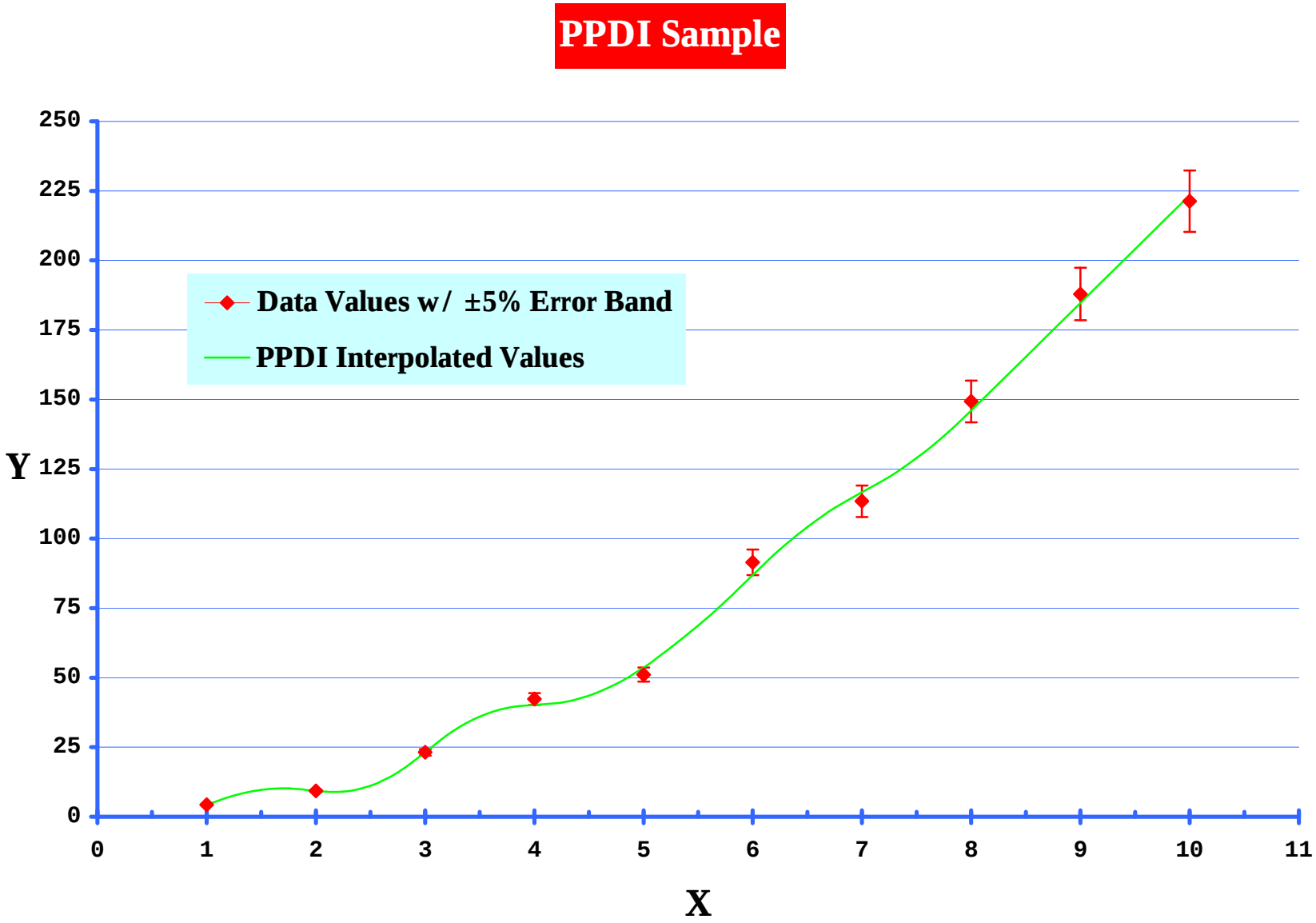
C A L C U L A T E D V A L U E S

Point	X	Y'	Point	X	Y'
1	1	4.290	6	6	86.906
2	2	9.300	7	7	116.639
3	3	23.170	8	8	146.006
4	4	40.185	9	9	184.715
5	5	53.665	10	10	223.424

rms Deviation = 2.606

I N T E R P O L A T E D V A L U E S

Point	X	Y'	Point	X	Y'
1	1.00	4.290	21	6.00	86.906
2	1.25	7.676	22	6.25	96.160
3	1.50	9.640	23	6.50	104.200
4	1.75	10.181	24	6.75	111.026
5	2.00	9.300	25	7.00	116.639
6	2.25	8.973	26	7.25	122.229
7	2.50	11.175	27	7.50	128.987
8	2.75	15.908	28	7.75	136.912
9	3.00	23.170	29	8.00	146.006
10	3.25	30.629	30	8.25	155.683
11	3.50	35.951	31	8.50	165.360
12	3.75	39.136	32	8.75	175.038
13	4.00	40.185	33	9.00	184.715
14	4.25	41.013	34	9.25	194.392
15	4.50	43.536	35	9.50	204.069
16	4.75	47.753	36	9.75	213.747
17	5.00	53.665	37	10.00	223.424
18	5.25	60.813			
19	5.50	68.735			
20	5.75	77.433			


PPDI Sample
Data Values w/ ±5% Error Band
PPDI Interpolated Values
Y
X
250
225
200
175
150
125
100
75
50
25
0
0
1
2
3
4
5
6
7
8
9
10
11